approach. A gain scheduling program would be a simple means of accomplishing this adaption.

4) As compared to the unalleviated or open loop configuration, utilizing optimal stochastic control theory and the root square locus concept resulted in an average 46% reduction in the mean square normal acceleration factor and an average 27% increase in the damping ratio of the short period roots.

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Center Deflections of Square Plates with Elastic Edge Beams

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Nomenclature

a	=	half plate width
D	=	flexural rigidity of a plate
E	=	modulus of elasticity of plate material
E_b	=	modulus of elasticity of edge beam material
EH	=	extensional rigidity of a plate
h	=	total plate thickness
I	=	moment of inertia of the edge cross-sectional area
N	=	load on edge beams
p	=	load on plate
t	=	face sheet thickness
U	=	edge beam displacement
U_o	=	edge beam midsection displacement
$\bar{u},\bar{v},\bar{w}$	=	displacement components of a membrane with rigid edge beams
W_L	=	largest transverse displacement
W_s	=	smallest transverse displacement
W_o	=	transverse displacement of the plate center
ν	=	Poisson's ratio

THE deformation of the side wall and its edge beams of an air plane cargo box during aircraft maneuvering may become excessive and intolerable. A rigorous analysis of the problem, which basically involves the determination of the deformation of plates supported by edge beams elastic in the plane of the plate, will be very difficult. An engineering approach for estimating the center deflections of a uniformly loaded square plate and the midsection deflection of the edge beam is presented. The side wall of a typical C-5A cargo box subjected to statically equivalent uniformly distributed inertia load during aircraft maneuvering is used as an example.

Received November 9, 1970. The work was a part of consulting work performed at the Lockheed-Georgia Company, Marietta, Ga.

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In this study, all materials are considered to be elastic, identical uniform edge beams are assumed to be perfectly rigid in the transverse direction and elastic in the plane of the plate, and only bending of edge beams in the plane of plate is considered. For uniformly loaded plate, the center deflection that represents the maximum deflection will be the main interest of this study.

The largest (upper bound) deflection w_L can be determined according to the bending theory of plate, i.e., when zero rigidity of edge beams in the plane of plate and no membrane effects are considered. The deflection of the plate may be generally represented as follows:

$$w_L = \alpha p(2a)^4/Df(x,y) \tag{1}$$

where a is the half plate width, p is the transverse loading, D is the plate rigidity, $f(x,y)$ represents the mode of deformation, and $f(0,0) = 1$. The coefficient α depends on the edge supporting conditions. According to Ref. 1 (see Tables 8 and 35),

$$\alpha = 0.00406 \text{ for simply supported plates} \tag{2a}$$

$$\alpha = 0.00126 \text{ for clamped plates} \tag{2b}$$

The maximum deflection is therefore

$$W_L = w_L(0,0) = \alpha p(2a)^4/D \tag{3}$$

The smallest (lower bound) deflection w_s can be determined by considering infinitely rigid edge beams and taking into account the membrane effects. The center deflection according to Ref. 1 (see pp. 422–424) may be obtained from the following equation:

$$p = [W_sD/\alpha(2a)^4] + W_s^3EH/0.516a^4 \tag{4}$$

where EH is the extensional rigidity. For a square plate with elastic edge beams as being considered in this study, the center deflection W_o will lie between W_L and W_s or

$$W_s < W_o < W_L \tag{5}$$

Square Plates with Elastic Edge Beams

Let U be the final deflection of the edge beam. The corresponding displacement W^* due to a portion p^* of the total load p, without consideration of the membrane effects is

$$w^* = \zeta W_of(x,y) = \alpha p^*(2a)^4/Df(x,y) \tag{6}$$

The function $f(x,y)$ satisfying the boundary conditions may be represented as follows:

$$f(x,y) = \cos\pi x/2a \cos\pi y/2a \tag{7a}$$

for a four sides simply supported plate, and

$$f(x,y) = \tfrac{1}{4}(1 + \cos\pi x/a)(1 + \cos\pi y/b) \tag{7b}$$

for a four sides clamped plate. The displacement U of an

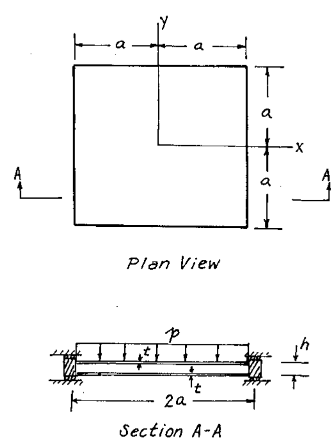

Fig. 1 Geometry and coordinates.

edge beam, say along $x = a$, is

$$U = \frac{1}{2}\int_0^a \left(\frac{\partial w^*}{\partial x}\right)^2 dx \tag{8}$$

By substituting Eq. (6) in conjunction with Eqs. (7a) and (7b) into Eq. (8), one obtains

$$U = \beta/4a[\pi\zeta W_o(1 + \cos\pi y/a)]^2 \tag{9}$$

The coefficient $\beta = \frac{1}{8}$ for simply supported plates and $\beta = \frac{1}{16}$ for clamped cases. The rest part of the total center displacement, $(1 - \zeta)W_o$, is caused by the load $\bar{p} = p - p^*$. The load $\bar{p}$ is balanced by bending, shearing, and membrane stresses. Let p_1 be the portion of $\bar{p}$ carried by bending and shearing stresses, and p_2 carried by membrane stresses. They are related to the center deflection as follows:

$$p_1 = D/\alpha(2a)^4(1 - \zeta)W_o \tag{10}$$

$$p_2 = EH/0.516\ a^4(1 - \zeta)^3W_o^3 \tag{11}$$

According to Ref. 1 (see p. 420) the deformation for a membrane with rigid boundaries are

$$\bar{u} = A(1 - \zeta)^2W_o\ /a\ \sin\pi x/a\ \cos\pi y/2a \tag{12}$$

$$\bar{v} = A(1 - \zeta)^2W_o^2/a\ \sin\pi x/a\ \cos\pi y/2a \tag{13}$$

$$\bar{w} = (1 - \zeta)W_o\ \cos\pi x/2a\ \cos\pi y/2a \tag{14}$$

For uniformly distributed load, with $\nu = 0.25$, the coefficient $A = 0.147$. The strain components of the plate at the edge beam location, say along $x = a$, are taken to be

$$\bar{\epsilon}_{xx} = \partial\bar{u}/\partial x + \tfrac{1}{2}(\partial\bar{w}/\partial x)^2$$

$$= \left[\frac{(1-\zeta)}{a}\ W_o\right]^2 \left[-0.147\pi\ \cos\frac{\pi y}{2a} + \frac{\pi^2}{8}\left(\cos\frac{\pi y}{2a}\right)^2\right] \tag{15}$$

$$\bar{\epsilon}_{yy} = (\partial\bar{v}/\partial y) + \tfrac{1}{2}(\partial\bar{w}/\partial y)^2 = 0 \tag{16}$$

The load N on the edge beam at $x = a$ becomes

$$N = EH\bar{\epsilon}_{xx} = K\left[-0.461\ \cos\frac{\pi y}{2a} + 0.615\left(1 + \cos\frac{\pi y}{a}\right)\right] \tag{17}$$

where

$$K = EH[(1 - \zeta/a)W_o]^2 \tag{18}$$

The displacement of edge beam along $x = a$ is governed by the equation

$$E_bId^4U/dy^4 = N \tag{19}$$

where I is the moment of inertia of the edge beam cross-sectional area. The solution of the beam clamped at both ends is

$$U = \frac{K}{E_bI}\left(\frac{a}{\pi}\right)^4 \left\{-7.376\ \cos\frac{\pi y}{2a} + 0.615\left[\frac{1}{24}\left(\frac{\pi y}{a}\right)^4 + \cos\frac{\pi y}{a}\right]\right\} + By^2 + C \tag{20}$$

where

$$B = -10.8EH/E_bI[(1 - \zeta)W_o/\pi^2]^2$$

$$C = 8.92EH/E_bI[(1 - \zeta)aW_o/\pi^2]^2$$

The beam midsection ($y = 0$) displacement becomes

$$U_o = 1.53EH/E_bI[(1 - \zeta)aW_o/\pi^2]^2 \tag{21}$$

By equating Eqs. (9) and (21), one obtains

$$\pi^2\zeta^2/16a = 1.53EH/E_bI[(1 - \zeta)a/\pi^2]^2 \tag{22}$$

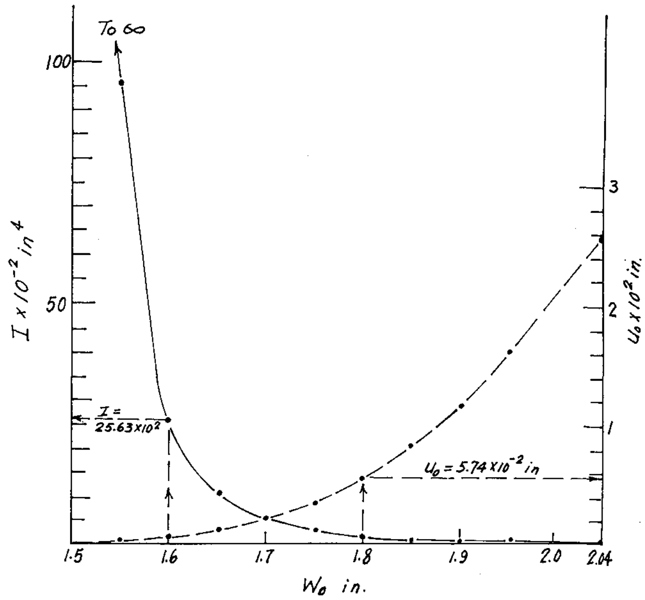

Fig. 2 Plate center displacement vs beam moment of inertia and midsection displacement.

By using Eqs. (6, 10, and 11), the relation between the total load p and the center plate displacement becomes

$$p = p^* + p_1 + p_2 = \frac{1}{a^4}\left[\frac{D}{16\alpha}W_o + \frac{EH}{0.506}(1 - \zeta)^3W_o^3\right] \tag{23}$$

Now the problem is completely solved. It is seen that if the material and dimensions of the plate and edge beams are given, one can determine ζ from Eq. (22). W_o and U_o can then be determined from Eqs. (23) and (21), respectively. On the other hand, if W_o is specified between W_s and W_L and wish to select an adequate edge beam, one can determine ζ from Eq. (23) first. I and U_o of the beam can then be determined from Eqs. (22) and (21), respectively.

Numerical Examples

A typical C-5A cargo box dimensions and a proposed wall construction shown in Fig. (1) are considered. The dimensions, loading and material constants are $a = 48$ in., $h = 1$ in., $p = 1$ psi, $E = 3 \times 10^6$ psi, $\nu = 0.25$, $E_b = 10 \times 10^6$ psi, and $t = 0.035$ in. The rigidities of the plate are found to be $D = 52.416 \times 10^3$ and $EH = 22.4 \times 10^4$. The plate is considered to be clamped along its four edges. The upper and lower bounds of the center deflections are $W_L = 2.042$ in. and $W_s = 1.5$ in. Numerical results describing the variation of I and U_o vs W_o are shown in Fig. 2. Detailed values and computer program can be found in Ref. 2.

Concluding Remarks

The paper presents an engineering method to estimate the maximum displacements of a uniformly loaded plate and its edge beams. Existing solutions for special cases are used in the proposed method. The solutions presented are not exact but are adequate for practical applications.

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